The resultant dipole moment of the combination

$$|\vec{p}_R| = 2p\cos\frac{\theta}{2} = 2p\cos\frac{120°}{2} = 2p\cos 60° = 2p\times\frac{1}{2} = p.$$

and the resultant dipole moment subtends an angle $\frac{\theta}{2} = 60°$ from either of two dipoles $\vec{p}_A$ or $\vec{p}_B$.

Therefore $\vec{p}_R$ subtends an angle 30° from +X direction.

If the system is subjected to electric field $\vec{E}$ directed along +X direction, the torque acting on the system is

$$\vec{\tau} = \vec{p}_R \times \vec{E}$$

Thus, the magnitude of torque is $|\vec{\tau}| = pE\sin 30° = \frac{1}{2}pE$ and the torque is directed into the plane of paper *i.e.*, the torque tends to align the system along the direction of electric field $\vec{E}$.

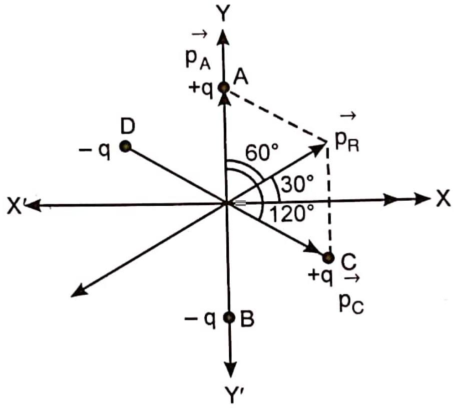

Fig. 1.51

(B – II) Short Answer Type Questions (3 marks each)

Q.34. Write Coulomb's law in vector form. Show that it is consistent with Newton's third law of motion.

Ans. Consider two point charges q_1 and q_2 separated by a distance r as shown in Fig. 1.52. Then force acting on charge q_1 due to q_2, according to Coulomb's law, is given by

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{21} \quad ...(i)$$

where $\hat{r}_{21}$ is a unit vector in the direction of q_1 from q_2 (or from q_2 to q_1).

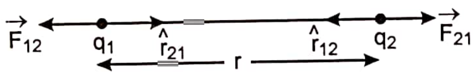

Fig. 1.52

Similarly force on charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad ...(ii)$$

where $\hat{r}_{12}$ is a unit vector from q_1 to q_2.

As unit vectors $\hat{r}_{21}$ and $\hat{r}_{12}$ have equal (unit) magnitude but are in mutually opposite directions, therefore,

$$\hat{r}_{21} = -\hat{r}_{12}$$

Hence, equation (*ii*) may be written as

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} = -\vec{F}_{21}$$

It shows that the forces exerted by the two charges on each other are equal and opposite. Thus, Coulomb's law is consistent with Newton's third law of motion.

Q.35. Define electric dipole moment. Is it a scalar or a vector ? Derive the expression for the electric field of a dipole at a point on the equatorial line of the dipole. [A.I. 2013]

The resultant dipole moment of the combination

$$|\vec{p}_R| = 2p \cos\frac{\theta}{2} = 2p\cos\frac{120°}{2} = 2p\cos 60° = 2p \times \frac{1}{2} = p.$$

and the resultant dipole moment subtends an angle $\frac{\theta}{2} = 60°$ from either of two dipoles $\vec{p}_A$ or $\vec{p}_B$.

Therefore $\vec{p}_R$ subtends an angle 30° from +X direction.

If the system is subjected to electric field $\vec{E}$ directed along +X direction, the torque acting on the system is

$$\vec{\tau} = \vec{p}_R \times \vec{E}$$

Thus, the magnitude of torque is $|\vec{\tau}| = pE \sin 30° = \frac{1}{2}pE$

and the torque is directed into the plane of paper *i.e.*, the torque tends to align the system along the direction of electric field $\vec{E}$.

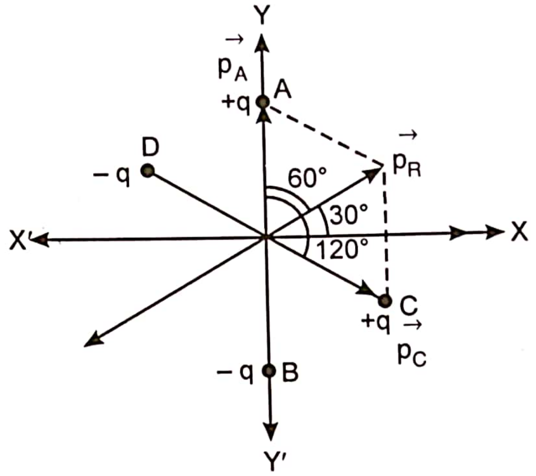

Fig. 1.51

(B – II) Short Answer Type Questions (3 marks each)

Q.34. Write Coulomb's law in vector form. Show that it is consistent with Newton's third law of motion.

Ans. Consider two point charges q_1 and q_2 separated by a distance r as shown in Fig. 1.52. Then force acting on charge q_1 due to q_2, according to Coulomb's law, is given by

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad ...(i)$$

where $\hat{r}_{21}$ is a unit vector in the direction of q_1 from q_2 (or from q_2 to q_1).

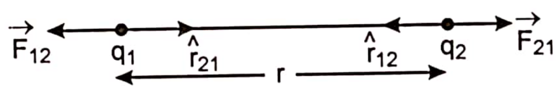

Fig. 1.52

Similarly force on charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad ...(ii)$$

where $\hat{r}_{12}$ is a unit vector from q_1 to q_2.

As unit vectors $\hat{r}_{21}$ and $\hat{r}_{12}$ have equal (unit) magnitude but are in mutually opposite directions, therefore,

$$\hat{r}_{21} = -\hat{r}_{12}$$

Hence, equation (*ii*) may be written as

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}\hat{r}_{12} = -\vec{F}_{21}$$

It shows that the forces exerted by the two charges on each other are equal and opposite. Thus, Coulomb's law is consistent with Newton's third law of motion.

Q.35. Define electric dipole moment. Is it a scalar or a vector ? Derive the expression for the electric field of a dipole at a point on the equatorial line of the dipole. **[A.I. 2013]**

Ans. For definition of dipole moment, see Point Number 31 under the heading "Chapter At A Glance."

Dipole moment is a vector. Let us calculate the electrostatic field at a point P on the equatorial line at a distance 'r' from mid-point O of an electric dipole AB.

Obviously, $|\vec{E}_A| = |\vec{E}_B| = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(a^2+r^2)}$

Resultant intensity at point P is $\vec{E} = \vec{E}_A + \vec{E}_B$.

Let us resolve $\vec{E}_A$ and $\vec{E}_B$ along and perpendicular to the dipole axis. We find that components $E_A\sin\theta$ and $E_B\sin\theta$ nullify each other and hence

$$|\vec{E}| = (E_A+E_B)\cos\theta = 2\cdot\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(a^2+r^2)}\cdot\frac{a}{\sqrt{a^2+r^2}}$$

$$= \frac{2qa}{4\pi\epsilon_0(a^2+r^2)^{3/2}} = \frac{p}{4\pi\epsilon_0(a^2+r^2)^{3/2}}$$

where $p = q.2a$ = dipole moment of electric dipole.

The direction of $\vec{E}$ is opposite to that of $\vec{p}$ *i.e.*, $\vec{E} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2+a^2)^{3/2}}$

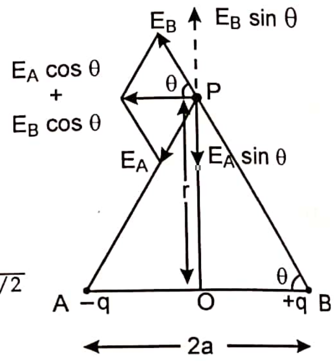

Fig. 1.53

If $r >> a$, then the above relation may be modified as

$$\vec{E} = -\frac{\vec{p}}{4\pi\epsilon_0 r^3}$$

Q.36. Derive an expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{p}$ kept in a uniform electric field $\vec{E}$. **[Delhi 2005, 2008, 2012 C, AI 2012 C, 2014]**

Ans. Consider an electric dipole AB placed in a uniform electric field $\vec{E}$ oriented at an angle θ with the field.

As shown in Fig. 1.54, forces qE and qE act on the two charges in mutually opposite directions. Consequently, net translational force on dipole is zero and there is no translatory motion of dipole.

However, as the two forces act at two different points non-linearly, they constitute a couple whose torque is given by :

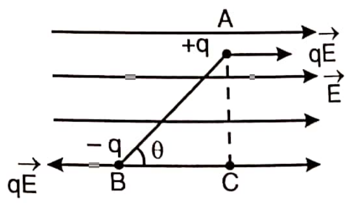

Fig. 1.54

torque $\tau = (qE)$. Normal distance between the forces

$= qE\,2a\sin\theta = pE\sin\theta$ $\quad [\because p = q(2a)]$

The torque has a tendency to align the dipole along the direction of electric field. Thus, according to right hand rule

$$\vec{\tau} = \vec{p}\times\vec{E}.$$

Q.37. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Obtain the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. **[Delhi 2015 C]**

Ans. The torque acting on the dipole $\vec{\tau} = \vec{p} \times \vec{E}$. For its derivation, see Answer to Short Answer Type Question Number 36.

Obviously, (*a*) $\vec{\tau}$ and $\vec{p}$, as well as (*b*) $\vec{\tau}$ and $\vec{E}$ are mutually perpendicular.

Q.38. Show mathematically that the electric field due to a short dipole at a distance '*d*' along its axis is twice the value of field at the same distance along the equatorial line.

Ans. We know that electric field at a distance '*d*' along the axis of an electric dipole is

$$E_{axial} = \frac{1}{4\pi \epsilon_0} \cdot \frac{2pd}{(d^2 - a^2)^2}$$

and for a short dipole ($a << d$), we have

$$E_{axial} = \frac{1}{4\pi \epsilon_0} \cdot \frac{2p}{d^3} \quad ...(i)$$

Again electric field at a distance '*d*' along the equatorial line of dipole is

$$E_{equatorial} = \frac{1}{4\pi \epsilon_0} \cdot \frac{p}{(d^2 + a^2)^{3/2}}$$

and in case of a short dipole

$$E_{equatorial} = \frac{1}{4\pi \epsilon_0} \cdot \frac{p}{d^3} \quad ...(ii)$$

Comparing (*i*) and (*ii*), we have

$$\frac{E_{axial}}{E_{equatorial}} = 2 \quad \text{or} \quad E_{axial} = 2 \times E_{equatorial}.$$

Q.39. Two identical pith balls, each of mass *m* and charge +*q*, are suspended from a point with threads of length *l* each. If in equilibrium state θ be the angle which each thread makes with vertical in equilibrium, find value of charge on each ball.

Ans. The equilibrium state is shown in following Fig. 1.55. Forces acting on either ball are

(*i*) Weight mg in vertically downward direction,

(*ii*) Coulombian force $F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{(AB)^2} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{4l^2 \sin^2\theta}$ in horizontal direction, and

(*iii*) Tension in thread T.

In equilibrium state $T\cos\theta = mg \quad ...(i)$

and $\quad T\sin\theta = F = \frac{q^2}{4\pi \epsilon_0 \cdot 4l^2 \sin^2\theta} \quad ...(ii)$

$$\Rightarrow \quad \tan\theta = \frac{q^2}{4\pi\epsilon_0 \cdot 4l^2 \sin^2\theta . mg}$$

or $\quad q = 2l \sin\theta . \sqrt{4\pi\epsilon_0 \cdot mg . \tan\theta}$.

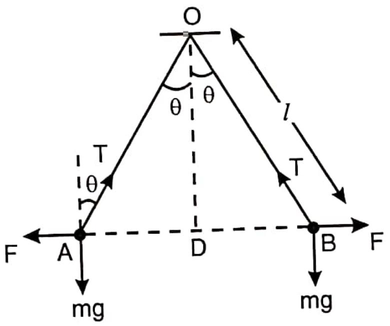

Fig. 1.55

Q.40. State Gauss's law in electrostatics. Using it derive an expression for the electric field due to an infinitely long straight wire of linear charge density λ C/m.
[A.I. 2005, 2007, Delhi 2004, 2009, 2011 C]

Or

State Gauss' law in electrostatics. A thin straight infinitely long conducting wire of linear charge density 'λ' is enclosed by a cylindrical surface of radius '*r*' and length '*l*', its axis coinciding with the length of the wire. Obtain the expression for the electric field, indicating its direction at a point on the surface of the cylinder. **[Delhi 2012 C]**

Ans. For statement of Gauss law in electrostatics, Refer to Point Number 50 under the heading "Chapter At A Glance".

Consider an infinitely long straight charged wire of linear charge density λ. To find electric field at a point P situated at a distance r from the wire by using Gauss' law consider a cylinder of length l and radius r as the Gaussian surface.

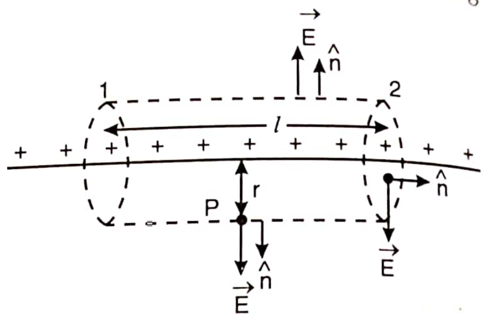

Fig. 1.56

From symmetry consideration electric field at each point of its curved surface is $\vec{E}$ and is pointed outwards normally. Therefore, electric flux over the curved surface

$$= \int \vec{E}.\hat{n}\, ds = E\, 2\pi r\, l$$

On the side faces 1 and 2 of the cylinder normal drawn on the surface is perpendicular to electric field E and hence these surfaces do not contribute towards the total electric flux.

$\therefore$ Net electric flux over the entire Gaussian surface $\phi_E = E.2\pi r\, l$ ...(*i*)

By Gauss law electric flux $\phi_E = \dfrac{1}{\epsilon_0}(\text{charge enclosed}) = \dfrac{\lambda l}{\epsilon_0}$ ...(*ii*)

Comparing (*i*) and (*ii*), we have

$$E.2\pi r\, l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi \epsilon_0 r}$$

As $E \propto \dfrac{1}{r}$, hence E-r graph is as shown in Fig. 1.57.

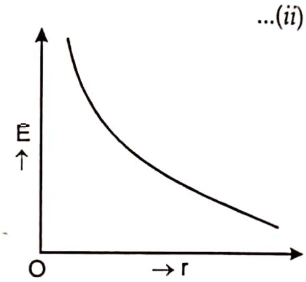

Fig. 1.57

Q.41. State Gauss's theorem in electrostatics. Using it, deduce an expression for the electric field due to a uniformly charged infinite plane thin sheet.

[Delhi 2000, 2006, 2007, 2009, A.I. 2004, 2005, 2007, 2016]

Or

Using Gauss' law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it. How is the field directed if the sheet is (*i*) positively charged, (*ii*) negatively charged.

[Delhi 2012]

Ans. For Gauss' theorem, see Point Number 50 under the heading "Chapter At A Glance".

Consider an infinite thin plane sheet having a surface density of charge σ. To find electric field at a point P situated at a normal distance r from the sheet, consider an imaginary cylinder of cross-section area ds around point P and length $2r$ passing through the sheet as the Gaussian surface.

From symmetry consideration, only side faces 1 and 2 of cylinder contribute towards the flux because here $\vec{E}$ and $\hat{n}$ are parallel but the curved surface of cylinder does not contribute towards the flux because here $\vec{E}$ and $\hat{n}$ are mutually perpendicular.

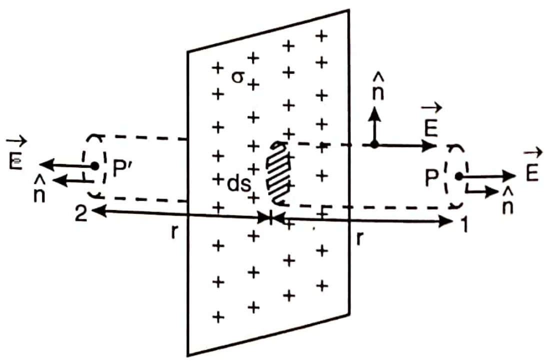

Fig. 1.58

$\therefore$ Total electric flux $\phi_E = 2E\,ds$ ...(*i*)

As per Gauss theorem total electric flux $\phi_E = \frac{1}{\epsilon_0}$(charge enclosed) $= \frac{1}{\epsilon_0}\cdot(\sigma\, ds)$ ...(*ii*)

Comparing (*i*) and (*ii*), we get

$$2\,E\,ds = \frac{\sigma}{\epsilon_0}\cdot ds$$

$$\Rightarrow \quad E = \frac{\sigma}{2\,\epsilon_0}$$

Thus, the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

Vectorially $\quad \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{r}$

Thus for (*i*) positively charged sheet electric field $\vec{E}$ is directed outwards and for (*ii*) negatively charged sheet the field is directed inwards.

Q.42. Use Gauss' law to derive the expression for the electric field between two uniformly charged large parallel sheets with surface charge densities σ and – σ respectively. [A.I. 2009]

Ans. First write the answer of Short Answer Type Question Number 41 given above. After that add the following :

Now consider two uniformly charged large parallel sheets *A* and *B* with surface charge densities σ and – σ respectively as shown in adjoining figure. At a point *P* lying between these two sheets.

$$|\vec{E}_A| = |\vec{E}_B| = \frac{\sigma}{2\,\epsilon_0}$$

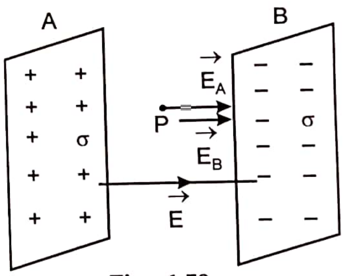

Fig. 1.59

Moreover, both $\vec{E}_A$ and $\vec{E}_B$ are directed in same direction. Hence, net electric field at point *P* will be

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

$$= \frac{\sigma}{\epsilon_0} \text{ normal to } A \text{ or } B \text{ towards } B.$$

Q.43. State Gauss law in electrostatics. Using this theorem derive the expression for the electric field at any point outside a uniformly charged thin spherical shell. [Delhi 2004, A.I. 2007]

Or

Using Gauss's theorem derive an expression for the electric field at any point outside a charged spherical shell of radius *R* and of charge density σ C/m^2. [A.I. 2006]

Ans. For Gauss' law, refer to Point Number 50 under the heading "Chapter At A Glance".

Consider a uniformly charged thin spherical shell of radius *R* and having a charge *Q*. To find electric field intensity at a point *P* outside the shell situated at a distance *r* (*r* > *R*) from the centre of shell, consider a sphere of radius *r* as the Gaussian surface. All points on this surface are equivalent relative to given charged shell and, thus, electric field $\vec{E}$ at all points of Gaussian surface has same magnitude *E* and $\vec{E}$ and $\hat{n}$ are parallel to each other.

∴ Total electric flux over the Gaussian surface

$$\phi_E = \oint \vec{E} \cdot \hat{n}\, ds = E.4\pi r^2 \qquad ..(i)$$

According to Gauss's theorem,

$$\phi_E = \frac{1}{\epsilon_0}(\text{charge enclosed}) = \frac{Q}{\epsilon_0} \qquad ...(ii)$$

Comparing (*i*) and (*ii*), we get

$$E.4\pi r^2 = \frac{Q}{\epsilon_0} \qquad \text{or} \qquad E = \frac{Q}{4\pi\epsilon_0 r^2}.$$

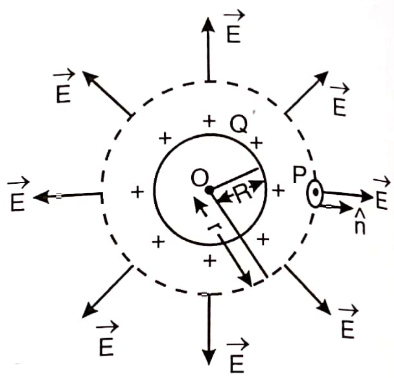

Fig. 1.60

From the result it is clear that for any point outside the shell, the effect is, as if whole charge Q is concentrated at the centre of the spherical shell.

If surface charge density of shell be σ then $Q = 4\pi R^2.\sigma$ and therefore,

$$E = \frac{4\pi R^2 \sigma}{4\pi\epsilon_0 r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

Q.44. State Gauss theorem in electrostatics. Prove that no electric field exists inside a hollow charged sphere. **[Delhi 2000, 2002, A.I. 2004, 2009]**

Ans. For Gauss theorem, refer to Point Number 50 under the heading "Chapter At A Glance".

Consider a hollow charged conducting sphere of radius R and having charge Q. To find electric field at a point P inside the shell, consider a sphere through point P and having centre O, *i.e.*, $r = OP$ (where $r < R$) as the Gaussian surface.

The electric flux through the Gaussian surface

$$\phi_E = \oint \vec{E}.\hat{n}\, ds = E\, 4\pi r^2 \qquad ...(i)$$

According to Gauss theorem, total electric flux should be

$$\phi_E = \frac{1}{\epsilon_0}(\text{charge enclosed}) = 0 \qquad ...(ii)$$

[Since the Gaussian surface is not enclosing any charge]

Thus, $\qquad \phi_E = E.4\pi r^2 = 0$

$\Rightarrow \qquad E = 0$

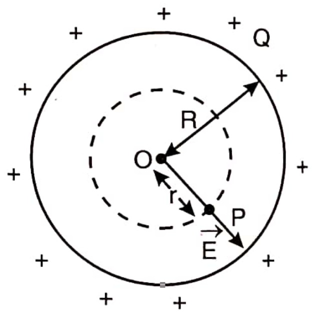

Fig. 1.61

Hence, no electric field exists inside a hollow charged sphere.

Q.45. Using Gauss's theorem show mathematically that for any point outside the shell, the field due to a uniformly charged thin spherical shell is the same as if the entire charge of the shell is concentrated at the centre. Why do you expect the electric field inside the shell to be zero according to this theorem ? **[A.I. 2006]**

Ans. To show that at any point outside the spherical shell electric field is same as if whole charge of shell is concentrated at the centre, see Short Answer Type Question Number 43.

At a point inside the shell, according to Gauss theorem, we expect the electric field to be zero because there is no charge present inside. Hence, electric flux and consequently electric field at that point will be zero.

Q.46. A thin conducting spherical shell of radius *R* has charge *Q* spread uniformly over its surface. Using Gauss' law, derive an expression for an electric field at a point outside the shell.

Draw a graph of electric field *E*(*r*) with distance '*r*' from the centre of the shell for $0 \le r \le \infty$.

[Delhi 2009]

Or

Using Gauss' law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r < R$ and $r > R$. **[Delhi 2011]**

Ans. For derivation of expression for an electric field at a point outside a charged thin conducting shell, see Short Answer Type Question Number 43.

Graph showing the variation of electric field $E(r)$ with distance r is shown in Fig. 1.62. For $r < R$, the electric field is zero and for $r > R$, the electric field $E \propto \frac{1}{r^2}$. Thus, field is maximum at the surface of the shell.

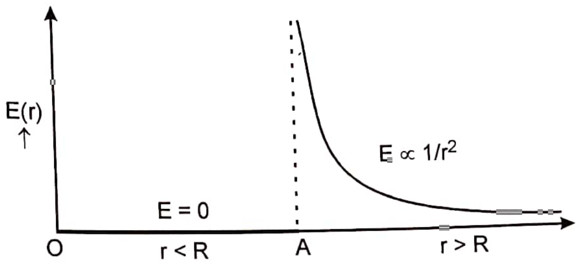

Fig. 1.62

Q.47. A positive point charge (+q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate.

Derive the expression for the electric field at the surface of a charged conductor. **[A.I. 2009]**

Ans. Electrical field lines have been shown in Fig. 1.63.

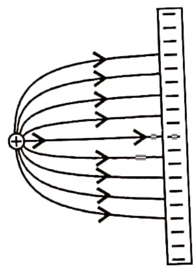

Fig. 1.63

Consider a charged conductor whose surface charge density is σ. To derive electric field at its surface consider a short cylinder (pill box) as the Gaussian surface about a given point. The cylinder is partly inside and partly outside the surface of the conductor. It has a small area of cross-section δS and negligible height.

Just inside the surface, the electric field is zero but just outside the field has a magnitude E and is directed normal to the surface. Thus, the contribution to the electric flux comes only from the outside circular cross-section of the cylinder.

$$\therefore \quad \phi_E = \vec{E}\cdot\vec{\delta S} = E\cdot\delta S \quad ...(i)$$

As per Gauss theorem :

$$\phi_E = \frac{1}{\epsilon_0}\,(\text{charge enclosed}) = \frac{1}{\epsilon_0}\cdot(\sigma\,\delta S) \quad ...(ii)$$

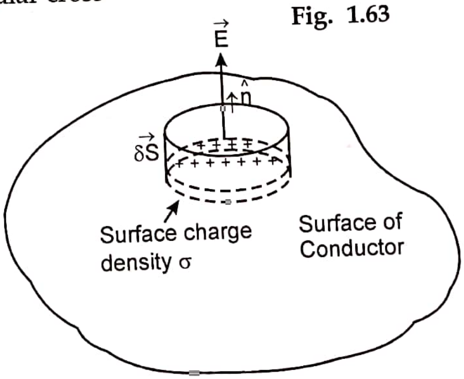

Fig. 1.64

Comparing (*i*) and (*ii*), we get

$$E = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \quad \vec{E} = \frac{\sigma}{\epsilon_0}\,\hat{n}$$

where $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Q.48. Show that the electric field at the surface of a charged conductor is given by $\vec{E} = \frac{\sigma}{\epsilon_0}\cdot\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. **[A.I. 2010]**

Ans. See the answer of Short Answer Type Question Number 47 given just above.

Q.49. A hollow conducting sphere of radius 8 cm is given a charge 16 μC. What is the electric field (*i*) at the centre of the sphere, and (*ii*) on the outer surface of the sphere ?

Ans. Here $Q = 16\ \mu C = 16 \times 10^{-6}$ C and $R = 8$ cm $= 0.08$ m.

(*i*) Electric field due to hollow sphere at its centre = 0, because field at any point inside a hollow sphere is always zero.

(*ii*) Electric field on the outer surface of the sphere is

$$E = \frac{Q}{4\pi\epsilon_0 R^2} = \frac{9 \times 10^9 \times 16 \times 10^{-6}}{(0.08)^2} = 2.25 \times 10^7 \text{ N C}^{-1}.$$

Q.50. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the Fig. 1.65. The electric field in the region is given by $\vec{E} = 50\, x\hat{i}$, where E is in N C^{-1} and x is in metres. Find

(*i*) Net flux through the cylinder.

(*ii*) Charge enclosed by the cylinder. **[Delhi 2013]**

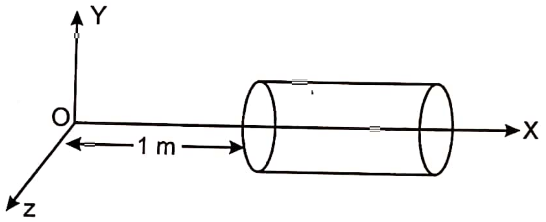

Fig. 1.65

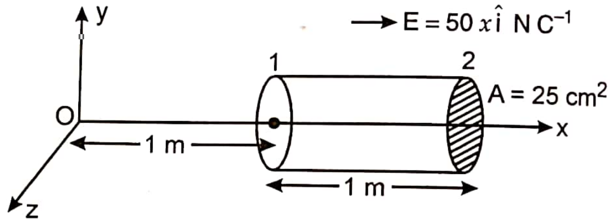

Fig. 1.66

Ans. (*i*) As electric field $\vec{E} = 50\, x\hat{i}$ N C^{-1} is along + ve direction of X-axis, hence flux passing through side faces 1 and 2 of cylinder is $\phi_1 = E_1 A$ inward and $\phi_2 = E_2 A$ outward. However, there is no flux associated with curved surface of cylinder.

$\therefore$ $\phi_1 = -E_1A = -(50 \times 1) \times 25 \times 10^{-4}$ N m^2 C^{-1} $= -0.125$ N m^2 C^{-1}

and $\phi_2 = +E_2A = +(50 \times 2) \times 25 \times 10^{-4}$ N m^2 C^{-1} $= +0.250$ N m^2 C^{-1}

$\therefore$ Net electric flux through the cylinder

$$\phi_E = \phi_1 + \phi_2 = -0.125 + 0.250 = +0.125 \text{ N m}^2 \text{ C}^{-1}$$

(*ii*) As net flux $\phi_E = \frac{q}{\epsilon_0}$, hence charge enclosed by the cylinder will be

$$q = \epsilon_0 \cdot \phi_E = (8.85 \times 10^{-12}) \times 0.125 = 1.1 \times 10^{-12} \text{ C} = 1.1 \text{ pC}$$

Q.51. Two point charges + q and – 2q are placed at the vertices B and C of an equilateral triangle ABC of side 'a' as shown in Fig. 1.67. Obtain the expression for (*i*) the magnitude, and (*ii*) the direction of the resultant electric field at the vertex A due to these two charges. **[Delhi 2014 C]**

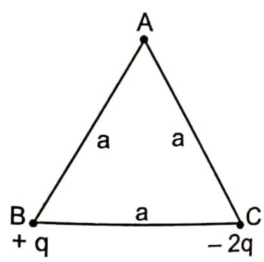

Fig. 1.67

Ans. As shown in Fig. 1.68, electric field at vertex A due to charge $+q$ placed at vertex B is

$$E_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} \text{ along } BA$$

and electric field at vertex A due to charge $-2q$ placed at vertex C is

$$E_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{a^2} \text{ along } AC$$

Obviously $\vec{E}_B$ and $\vec{E}_C$ are inclined at an angle $\theta = 120°$ from one another. Hence (*i*) Magnitude of net electric field at A

$$E = \sqrt{E_B^2 + E_C^2 + 2E_B E_C \cos 120°} = \sqrt{E_B^2 + E_C^2 - E_B E_C}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sqrt{3}q}{a^2}$$

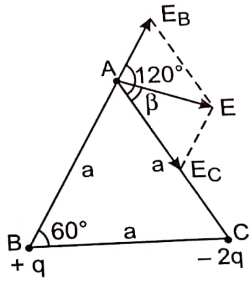

Fig. 1.68

(*ii*) If the net electric field $\vec{E}$ is directed at angle β from the direction AC, then

$$\tan\beta = \frac{E_B \sin 120°}{E_C + E_B \cos 120°} = \frac{E_B \cdot (\sqrt{3}/2)}{E_C - (E_B/2)} = \frac{E_B \cdot (\sqrt{3}/2)}{2E_B - (E_B/2)}$$

$$\Rightarrow \quad \tan\beta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \beta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Q.52. Two point charges q_1 and q_2 of magnitude $+10^{-8}$ C and -10^{-8} C respectively are placed 0.1 m apart. Calculate the electric field at points B, C and A (refer to Fig. 1.69). **[NCERT Exemplar]**

Ans. Given charges q_1 and q_2 constitute an electric dipole of dipole moment $p = q.2a = 10^{-8} \times 0.1 = 10^{-9}$ C-m

(*i*) Point B lies on the axial line of dipole, where $r = 0.05 + 0.05 = 0.1$ m, hence

$$E_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-9} \times 0.1}{[(0.1)^2 - (0.05)^2]^2}$$

$= 3.2 \times 10^4$ N/C directed along NM.

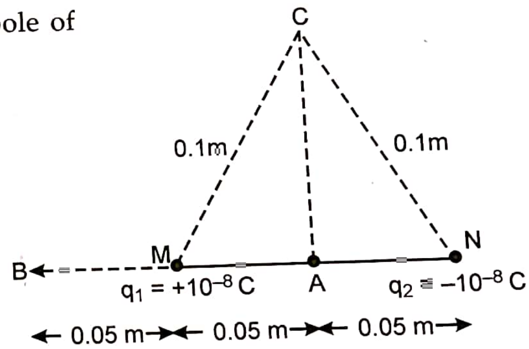

Fig. 1.69

(*ii*) Point C lies on the equatorial line of dipole, where $\sqrt{a^2 + r^2} = 0.1$ m, hence

$$\therefore \quad E_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(a^2 + r^2)^{3/2}} = \frac{9 \times 10^9 \times 10^{-9}}{(0.1)^3} = 9 \times 10^3 \text{ N/C along } MN.$$

(*iii*) Point A also lies at equatorial line where $r = 0$

$$\therefore \quad E_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{a^3} = \frac{9 \times 10^9 \times 10^{-9}}{(0.05)^3} = 7.2 \times 10^4 \text{ N/C along } MN.$$

Q.53. Charges of + 2, + 2 and – 2 μC, respectively are placed at the vertices of an equilateral triangle of side 0.3 m each in Fig. 1.70. Find net force experienced by each charge. **[NCERT Exemplar]**

Ans. Here $AB = AC = BC = 0.3$ m and $q_A = q_B = +2\ \mu C = 2 \times 10^{-6}$ C and $q_C = -2\ \mu C = -2 \times 10^{-6}$ C. Thus, it is clear that mutual force of attraction/repulsion between any two charges have equal magnitude given by

$$F = |\vec{F}_{AB}| = |\vec{F}_{AC}| = |\vec{F}_{BA}| = |\vec{F}_{BC}|$$

$$= |\vec{F}_{CA}| = |\vec{F}_{CB}| = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{(0.3)^2} = 0.4 \text{ N}$$

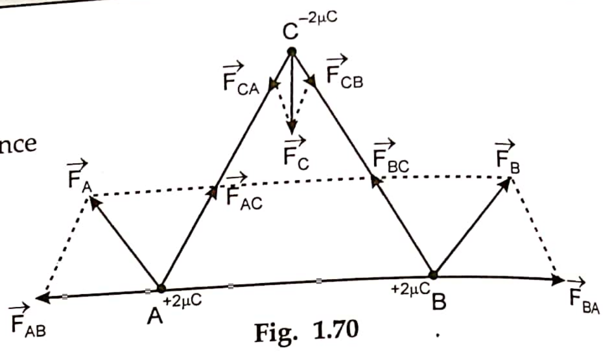

Fig. 1.70

Direction of these forces have been shown in Fig. 1.70.

(*a*) As angle between $\vec{F}_{CA}$ and $\vec{F}_{CB}$ is 60°, hence

$$|\vec{F}_C| = 2F_{CA}\cos\left(\frac{60}{2}\right)^\circ$$

$$= 2 \times 0.4 \times \frac{\sqrt{3}}{2}$$

$= 0.69$ N and the force is $\perp$ to AB.

(*b*) As angle between $\vec{F}_{AB}$ and $\vec{F}_{AC}$ is 120°, hence

$$|\vec{F}_A| = 2F_{AB}\cos 60° = 2 \times 0.4 \times \frac{1}{2} = 0.4 \text{ N}$$

and the force makes an angle 60° from line AC.

(*c*) As in (*b*) force $|\vec{F}_B| = 0.4$ N inclined at an angle of 60° from line BC.

Q.54. Two large metal plates of area 6.0 m^2 face each other. The plates are 3 cm apart and carry equal and opposite charges on their inner surfaces. If electric field at a point between the plates is 5×10^4 N C^{-1}, then calculate the magnitude of charge on each plate.

Ans. Let magnitude of charge on the inner surface of each plate be Q.

It is given that $A = 6.0$ m^2 and $E = 5 \times 10^4$ N C^{-1}

At a point P between the two plates, electric field due to both plates is equal and directed in same direction, *i.e.*,

$$\vec{E}_1 = \vec{E}_2 = \frac{\sigma}{2\epsilon_0} \quad \text{[From plate 1 towards plate 2]}$$

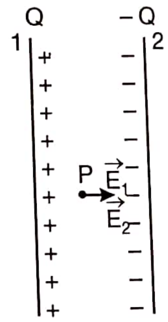

Fig. 1.71

$\therefore$ Net electric field at point P

$$E = E_1 + E_2 = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Rightarrow \quad Q = \epsilon_0 A E = 8.85 \times 10^{-12} \times 6.0 \times 5 \times 10^4$$

$= 2.655 \times 10^{-6}$ C or 2.66 μC.

Q.55. If the electric field is given by $6\hat{i} + 3\hat{j} + 4\hat{k}$, calculate the electric flux through a surface of area 20 units lying in *y-z* plane.

Ans. Since area $A = 20$ units lie in *y-z* plane, hence $\vec{A} = 20\hat{i}$

As $\vec{E} = 6\hat{i} + 3\hat{j} + 4\hat{k}$ is a uniform electric field, hence

Total electric flux $\phi_E = \vec{E}.\vec{A} = (6\hat{i} + 3\hat{j} + 4\hat{k}).(20\hat{i}) = 120$ units.

Q.56. A tiny particle of mass 5 μg is kept over a large horizontal sheet of charge density 4×10^{-6} C m^{-1}. What charge should be given to the particle so that if released it does not fall down ?

Ans. Let a charge q be given to the particle. As particle does not fall, hence downward weight of particle mg = upward force due to electric field of charged plate $qE = q\cdot\frac{\sigma}{2\epsilon_0}$.

$$\therefore \quad q = \frac{2mg\epsilon_0}{\sigma}$$

Here, $m = 5\ \mu g = 5\times10^{-6}\ g = 5\times10^{-9}$ kg, $\sigma = 4\times10^{-6}\ C\ m^{-1}$

$$\therefore \quad q = \frac{2\times5\times10^{-9}\times9.81\times8.85\times10^{-12}}{4\times10^{-6}} = 2.17\times10^{-13}\ C.$$

Q.57. A small metal sphere carrying charge +Q is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell. Use Gauss law to find the expression for the electric field (*i*) at a point P_1 situated inside the cavity at a distance x_1 from centre, (*ii*) at a point P_2 situated in the metallic spherical shell at a distance x_2 from the centre. [A.I. 2012 C]

Ans. Let a charge $+Q$ is located at the centre O of a spherical cavity inside a large uncharged metallic spherical shell as shown in Fig. 1.72. Then a charge $-Q$ is induced on inner surface of shell and a charge $+Q$ is induced on the outer surface of the shell.

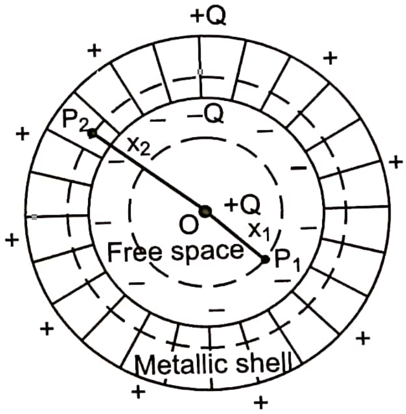

Fig. 1.72

(*i*) Let P_1 be a point inside the cavity at a distance x_1 from the centre O. To find electric field E_1 at point P_1 let us consider a spherical surface of radius x_1 as the Gaussian surface. Then, electric flux

$$\phi_E = \int \vec{E}\cdot\vec{dS} = E_1 s_1 = E_1(4\pi x_1^2)$$

$$= \frac{1}{\epsilon_0}(\text{charge enclosed}) = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \quad E_1 = \frac{Q}{4\pi\epsilon_0 x_1^2}$$

(*ii*) Let P_2 be a point situated in the metallic shell at a distance x_2 from the centre. To find electric field E_2 at point P_2 let us again consider a spherical surface of radius x_2 as the Gaussian surface. Then Electric flux

$$\phi_E = E_2 s_2 = E_2(4\pi x_2^2) = \frac{1}{\epsilon_0}(\text{charge enclosed}) = \frac{1}{\epsilon_0}(Q-Q) = 0$$

$$\Rightarrow \quad E_2 = 0$$

Q.58. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field (*a*) inside the sphere, (*b*) just outside the sphere, and (*c*) at a point 18 cm from the centre of the sphere ?

Ans. Here $q = 1.6\times10^{-7}$ C and $R = 12$ cm $= 0.12$ m

(*a*) Electric field intensity at any point inside the sphere $E = 0$.

(*b*) Electric field intensity at a point just outside the sphere

$$E = \frac{q}{4\pi\epsilon_0\cdot R^2} = \frac{1.6\times10^{-7}\times9\times10^9}{(0.12)^2} = 10^5\ N\ C^{-1}.$$

(*c*) Electric field intensity at a point 18 cm from the centre of sphere (*i.e.*, $r = 18$ cm $= 0.18$ m)

$$E = \frac{q}{4\pi\epsilon_0\cdot r^2} = \frac{1.6\times10^{-7}\times9\times10^9}{(0.18)^2} = 4.44\times10^4\ N\ C^{-1}.$$

Q.59. An early model for an atom considered it to have a positively charged point nucleus of charge +Ze surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus when (i) r < R, (ii) r = R and (iii) r > R ? Use Gauss's theorem. **[NCERT Exemplar]**

Ans. Let as shown in Fig. 1.73 positive charge at the centre of the atom be +Ze and let density of negative charge be ρ, such that total negative charge $\frac{4}{3}\pi R^3\rho = -Ze$

$$\Rightarrow \qquad \rho = -\frac{3Ze}{4\pi R^3}$$

(i) For any point P situated at a distance r (r < R) from the nucleus considering a sphere of radius r as the Gaussian surface, we have

Charge enclosed

$$q = (+Ze) + \text{negative charge enclosed within the sphere of radius } r$$

$$\Rightarrow \qquad q = Ze + \frac{4}{3}\pi R^3\rho = Ze - \frac{Zer^3}{R^3} = Ze\left[1 - \frac{r^3}{R^3}\right]$$

$$\therefore \text{ Flux on Gaussian surface } \phi_E = E.4\pi r^2 = \frac{1}{\epsilon_0}q = \frac{Ze}{\epsilon_0}\left[1 - \frac{r^3}{R^3}\right]$$

$$\Rightarrow \qquad E = \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]$$

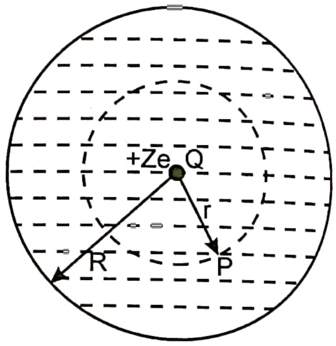

Fig. 1.73

(ii) For any point P situated on or outside the atom (i.e., r ≥ R), total charge enclosed

$$q = (+Ze) + (-Ze) = 0$$

Hence $\qquad \phi_E = E.4\pi r^2 = \frac{1}{\epsilon_0}(0) = 0$

$$\Rightarrow \qquad E = 0.$$

Q.60. Two identical metallic spheres A and B having charges +4Q and –10Q are kept a certain distance apart. A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B. Spheres A and B are then brought in contact and then separated. Find the charges on the spheres A and B. **[A.I. 2011 C]**

Ans. Initial charge on sphere A, $q_A = +4Q$ and charge on B, $q_B = -10Q$. When an identical uncharged sphere C is placed in contact with sphere A, it shares its charge and charge on C, $q_C = \frac{q_A}{2} = +\frac{4Q}{2} = 2Q$ and the remaining charge on A becomes $q'_A = +2Q$.

Now sphere C having charge $q_C = +2Q$ is brought in contact with sphere B having charge $q_B = -10Q$. The two spheres share their charges and acquire charges

$$q'_C = q'_B = \frac{(+2Q - 10Q)}{2} = -4Q$$

Now finally spheres A (charge $q'_A = +2Q$) and B (charge $q'_B = -4Q$) are brought in contact and then separated. On contact A and B share their charges equally and final value of charge on A or B will be

$$q''_A = q''_B = \frac{+2Q - 4Q}{2} = -Q$$

(C) Long Answer Type Questions (5 marks each)

Q.1. What is meant by continuous charge distribution ? How do you apply superposition principle to obtain total force on a point charge due to all the three types of a continuous charge distribution ?

Ans. A continuous charge distribution may be means a system of extra large number of closely spaced charges. A continuous charge distribution either (*i*) a linear charge distribution, or (*ii*) a surface charge distribution, or (*iii*) a volume charge distribution. Let us calculate force on a point charge due to these charge distributions one by one :

(*i*) *Linear charge distribution* in which charge is distributed uniformly along a line *e.g.*, a charged wire or ring etc.

Let there be a linear charge distribution having linear charge density λ. If we consider a point charge q_0 at point P situated at a distance r from a small element of length dl, then force due to this element will be

$$d\vec{F} = \frac{q_0}{4\pi\epsilon_0}\cdot\frac{\lambda\, dl}{r^2}\hat{r}$$

and hence $$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_l \frac{\lambda\, dl}{r^2}\hat{r}$$

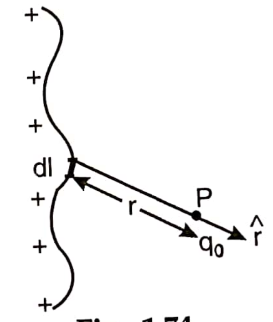

Fig. 1.74

(*ii*) *Surface charge distribution* in which charge is distributed continuously over some area *e.g.*, a charged thin sheet. Let there be a surface charge distribution having surface charge density σ. When we consider a point charge q_0 at point P situated at a distance r from a small surface element of area ds, then

$$d\vec{F} = \frac{q_0}{4\pi\epsilon_0}\cdot\frac{\sigma . ds}{r^2}\hat{r}$$

and hence $$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_S \frac{\sigma\, ds}{r^2}\hat{r}$$

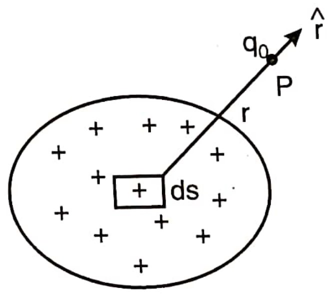

Fig. 1.75

(*iii*) *Volume charge distribution* in which charge is distributed continuously over a volume *e.g.*, a metal sphere or cylinder etc.

Let charge density be ρ. Then force on a point charge q_0 due to a charge element of volume dV will be

$$d\vec{F} = \frac{q_0}{4\pi\epsilon_0}\cdot\frac{\rho\, dV}{r^2}\hat{r}$$

and hence $$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_V \frac{\rho\, dV}{r^2}\hat{r}$$

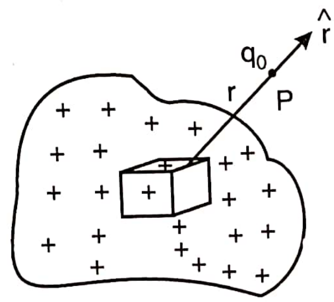

Fig. 1.76

In general, if all the three types of continuous charge distributions are present then net force experienced by a given point charge q_0 will be given by

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\left[\int_l \frac{\lambda\, dl}{r^2}\hat{r} + \int_S \frac{\sigma\, ds}{r^2}\hat{r} + \int_V \frac{\rho\, dV}{r^2}\hat{r}\right]$$

Q.2. (*a*) **Define electric flux. Write its SI units.**

(*b*) **The electric field components due to a charge inside the cube of side 0.1 m area as shown in Fig. 1.77 : $E_x = \alpha x$, where $\alpha = 500$ N/C m, $E_y = 0$ and $E_z = 0$.** **[A.I. 2008]**

Calculate (*i*) the flux through the cube, and (*ii*) the charge inside the cube.

Ans. (*a*) See Point Numbers 45 and 46 under the heading "Chapter At A Glance".

(b) (i) As E_y and E_z are zero and $E = E_x = \alpha x$, hence electric flux is linked only with two faces of the cube lying in y-z plane (*i.e.*, perpendicular to $\vec{E_x}$).

At the position of left face of cube $x = 0.1$ m, hence $E_x = \alpha x = 500 \times 0.1 = 50$ N/C

and surface area of face $s = (0.1)^2 = 0.01\ m^2$

$\therefore$ Flux on this face $\phi_1 = E_x s = 50 \times 0.01 = 0.5\ N\ m^2\ C^{-1}$ (inward)

Again on the opposite face of cube (*i.e.*, right face) $x = 0.2$ m and

hence $E'_x = \alpha x = 500 \times 0.2 = 100$ N/C

$\therefore$ Flux on this face $\phi_2 = E'_x \cdot s = 100 \times 0.01 = 1\ N\ m^2\ C^{-1}$ (outward)

$\therefore$ Net electric flux through the cube $\phi = 0.5\ N\ m^2\ C^{-1}$ (inward) $+ 1.0\ N\ m^2\ C^{-1}$ (outward)

$= +0.5\ N\ m^2 C^{-1}$ (outward)

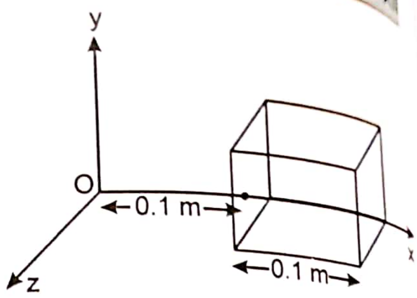

Fig. 1.77

(ii) From Gauss's law $\phi = \dfrac{q}{\epsilon_0}$

$\Rightarrow$ Charge inside the cube $q = \phi \cdot \epsilon_0 = +0.5 \times 8.85 \times 10^{-12}$

$= +4.42 \times 10^{-12}$ C

Q.3. (a) An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance '2a' apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit $r >> a$, $\vec{E} = \dfrac{2\vec{p}}{4\pi \epsilon_0 r^3}$.

(b) Given the electric field in the region $\vec{E} = 2x\hat{i}$ [refer to Fig. 1.78], find the net electric flux through the cube and the charge enclosed by it. **[Delhi 2015]**

Ans. (a) See Short Answer Type Question Number 15.

(b) As electric field is given by $\vec{E} = 2x\hat{i}$, it is obvious that electric flux $\phi_E = \int \vec{E} \cdot \vec{ds}$ is finite for only two surfaces marked 1 and 2 in figure, which lie in Y-Z plane and for all remaining four surfaces of cube, flux is zero.

Here area of each surface $s = a^2$.

For face 1, $x = 0$ and $\vec{E} = 0$. Therefore electric flux $\phi_1 = 0$.

For face 2, $x = a$ and $\vec{E} = 2a\hat{i}$. Therefore electric flux

$$\phi_2 = \vec{E} \cdot \vec{s} = (2a\hat{i}) \cdot (a^2\hat{i}) = 2a^3$$

$\therefore$ Net electric flux through the cube $\phi_E = \phi_1 + \phi_2 = 0 + 2a^3 = 2a^3$

and the charge enclosed by the cube $q = \epsilon_0 \cdot \phi_E = \epsilon_0 \cdot 2a^3 = 2\epsilon_0 a^3$

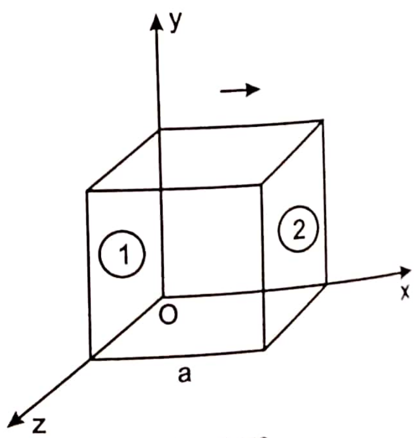

Fig. 1.78

Q.4. ***(a)*** **Using Gauss's law derive an expression for the electric field at any point outside a uniformly charged thin spherical shell of radius *R* and charge density σ C/m². Draw the field lines when the charge density of the sphere is *(i)* positive *(ii)* negative.**

(b) **A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m². Calculate the**

(i) **charge on the sphere, *(ii)* total electric flux passing through the sphere.** **[Delhi 2008]**

Ans. *(a)* See Short Answer Type Question Number 43.

For electric field lines, see Figs. 1.18 and 1.19 respectively.

(b) As diameter $D = 2.5$ m, hence radius $R = \frac{D}{2} = 1.25$ m

(i) ∴ Total charge on the sphere $Q = \sigma . 4\pi R^2$

$$= (100\ \mu C/m^2) \times 4\pi \times (1.25)^2\ m^2 = 1.96 \times 10^{-3}\ C$$

(ii) Total electric flux through the sphere

$$\phi_E = \frac{1}{\epsilon_0} \cdot (Q) = \frac{1.96 \times 10^{-3}}{8.85 \times 10^{-12}} = 2.2 \times 10^8\ V\ m.$$

Q.5. State Gauss' law in electrostatics.

Consider an overall neutral sphere of radius *R*. This sphere has a point charge +*Q* at its centre and this positive charge is surrounded by a uniform density ρ of negative charge up to a radius *R*.

Use Gauss' law to obtain expressions for the electric field, of this sphere, at a point distant *r*, from its centre, where *(i)* *r* < *R*, *(ii)* *r* > *R*.

Show that these two expressions give identical results, for the electric field, at *r* = *R*.

[A.I. 2010 C]

Ans. For statement of Gauss' law, see Point Number 50 under the heading "Chapter At A Glance".

Consider a sphere of radius R and having a point charge $+Q$ at its centre point. If negative charge density of sphere be ρ, then total negative charge on sphere $= \frac{4}{3}\pi R^3 \cdot \rho = -Q$

$$\Rightarrow \quad \rho = -\frac{3Q}{4\pi R^3}$$

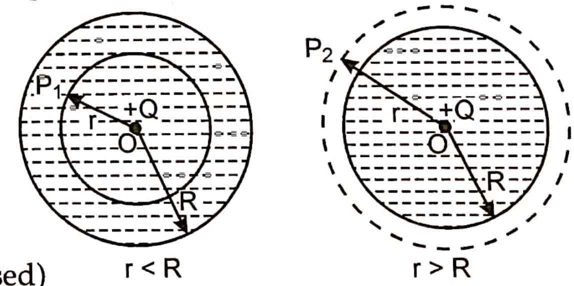

Fig. 1.79

(i) For a point P_1 situated at a distance r (where $r < R$) from its centre, considering a sphere of radius 'r' as the Gaussian surface, we have

$$\Rightarrow \quad \phi_E = \oint \vec{E} \cdot \vec{ds} = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}(\text{charge enclosed})$$

$$= \frac{1}{\epsilon_0}\left[Q + \rho \cdot \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \quad E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\left[Q - \frac{3Q}{4\pi R^3} \cdot \frac{4}{3}\pi r^3\right] = \frac{1}{\epsilon_0}\left[Q - \frac{Qr^3}{R^3}\right] = \frac{Q}{\epsilon_0}\left[1 - \frac{r^3}{R^3}\right]$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]$$

For $r = R$, we have $E_{surface} = 0$.

(ii) For a point P_2 situated at a distance r (where $r > R$) we again consider a sphere of radius r as the Gaussian surface. Now total charge enclosed in the surface $= Q + \rho \cdot \frac{4}{3}\pi R^3 = Q - \frac{3Q}{4\pi R^3} \cdot \frac{4}{3}\pi R^3$

$= Q - Q = 0.$

This shows that $\phi_E = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}(\text{charge enclosed}) = \frac{1}{\epsilon_0}(0) = 0$

$\Rightarrow \quad E = 0$

and field at surface of sphere $E_{surface} = 0$

Q.6. Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside, and (ii) inside the shell.

Plot a graph showing variation of electric field as a function of r for $r > R$ and $r < R$. (r being the distance from the centre of the shell). **[A.I. 2011 C, 2013, 2013 C]**

Ans. For expression of electric field at a point outside the shell, see Short Answer Type Question Number 43 and for field at a point inside the shell, see Short Answer Type Question Number 44.

The plotted E-r graph has been shown in Fig. 1.62.

Q.7. (a) Define electric flux. Write its SI unit.

"Gauss' law in electrostatics is true for any closed surface, no matter what its shape or size is." Justify this statement with the help of a suitable example.

(b) Use Gauss' law to prove that the electric field inside a uniformly charged spherical shell is zero. **[A.I. 2015]**

Ans. (a) For electric flux and its SI unit, see Point Number 45 and 46 under the heading "Chapter At A Glance".

To justify the statement that Gauss' law is true for any closed surface irrespective of its shape or size, let us consider two spherical surfaces of radii r_1 and r_2 respectively with a charge q at its centre.

Now electric field at the surface of 1st sphere $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1^2}$

normally outward and flux over this surface

$$\phi_1 = \int \vec{E} \cdot \vec{ds} = E_1 s_1 = \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1^2}\right) \times (4\pi r_1^2) = \frac{q}{\epsilon_0}$$

Again electric field at the surface of 2nd sphere $E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_2^2}$

normally outward and flux over this surface

$$\phi_2 = \int \vec{E} \cdot \vec{ds} = E_2 s_2 = \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_2^2}\right) \times (4\pi r_2^2) = \frac{q}{\epsilon_0}$$

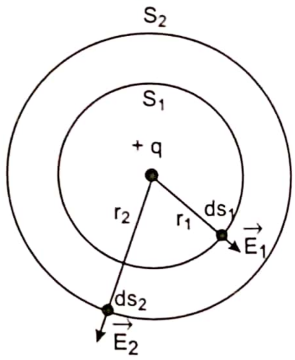

Fig. 1.80

Thus, it is clear that $\phi_1 = \phi_2$, *i.e.*, electric flux is same for both the closed surfaces.

(b) See Short Answer Type Question Number 44.

PROBLEMS FOR PRACTICE

Q.1. *How far apart two protons be if the electrostatic force exerted by one on the other is equal to weight of the electron ?*

[Hint : Here $q_1 = q_2 = +1.60 \times 10^{-19}$ C and $F = m_e g = 9 \times 10^{-31} \times 9.81$ N] **[Ans.** 5.1 m]

Q.2. *Two point electric charges $+q$ and $+4q$ are separated by a distance of $6a$. Find the point on the line joining the two charges where the electric field is zero.* **[Ans.** $2a$ from charge q or $4a$ from charge $4q$]